PYRAMID Technical Consultants, Inc.

Technical Note

TN0001
Some useful formulae for high energy ion beam systems

Rest mass of particle in eV

$$U_0 = \frac{auc^2}{e} = 9.3149433 \times 10^8 a$$

a is particle mass in amu, u: atomic mass unit = 1.6605389e-27 kg, c is speed of light in vacuum = 299 792 458 m s-1, e is elementary charge unit = 1.60217657e-19 C. Divide U_0 by 1e6 for value in MeV or by 1e9 for value in GeV.

Gamma, beta and velocity

$$\gamma = \frac{U_T}{U_0} = \frac{U_0 + U_K}{U_0}$$

U_T is total particle energy, U_K is particle kinetic energy

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$v = \beta c$$

v is velocity in m s-1.

Closed orbits

$$f_O = \frac{v}{C} = \frac{\beta c}{C}$$

f_0 is the orbit frequency in Hz, C is the orbit circumference (not necessarily circular) in m. Note that the RF frequency in a closed orbit accelerator may be a harmonic of the orbit frequency. The particle kinetic energy can be recovered from the orbit frequency:

$$\frac{U_K}{U_0} = \sqrt{\frac{1}{1 - \left(\frac{f_0 C}{c}\right)^2}} - 1$$

Momentum

$$p = \beta\gamma U_0$$

p is momentum in units of eV/c, MeV/c or GeV/c according to the units of U_0. To express in SI units (kg m s-1), multiply by 5.34429e-22.

Solving for kinetic energy in terms of momentum and rest mass

$$U_k = \sqrt{p^2 + U_0^{\ 2}} - U_0$$

Magnetic rigidity

$$B\rho = \frac{p}{qc}$$

Bρ is magnetic rigidity of the particle in Tm (telsa-meters), p is momentum in eV/c, q is the charge state of the ion (q=1 for protons).

Bending radius in a magnetic field

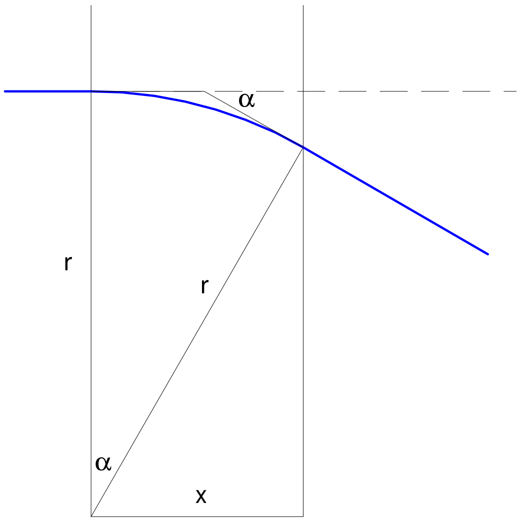
$$r = \frac{B\rho}{B}$$

r is the gyromagnetic radius in m, B is the field in T.

Deflection in a parallel straight edged uniform magnetic field with sharp cut off

$$\alpha = \pi/2 - \cos^{-1}\left(\frac{x}{r}\right)$$

α is deflection angle in radians, x is the distance between the field edges in m. For small angles (< 10 degrees), the intersection point between the entrance ray and the exit ray (the nominal instantaneous deflection point) can be assumed to be equidistant from the entrance and exit faces to good approximation.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